**Spin chromaticity of beam: orbit lengthening and betatron chromaticity**

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***Abstract***

One possible method of measuring the electric dipole moment of an elementary particle consists in measuring the average spin precession frequency of a polarized beam. The problem of maximizing the spin coherence time of a bunched beam is reduced to the minimization of the particles’ orbit length dispersion. This is achieved by introducing sextupoles with fields minimizing the chromaticity of the spin frequency. We have found experimentally that minimizing the chromaticity of betatron motion leads to the minimization of the chromaticity of the spin frequency. In this paper we explore this relationship.

Key words: orbit lengthening, spin, betatron oscillation, chromaticity

**Introduction**

The main idea of ​​measuring the electric dipole moment is based on measuring the averaged precession frequency of the spin of particles in a bunched beam; hence, the time during which the particle spin oscillations remain coherent plays a decisive role in the experiments searching for the electric dipole moment of the proton/deuteron. This time is called the “spin coherence time,” and it must be prolonged as long as possible. Spin decoherence effects arise due to the dependence of the spin frequency on particle oscillations in three-dimensional space. Since all particles remain within the separatrix during circulation, their cyclotron frequencies must be equal regardless of orbit-length. In view of this fact the so-called “effective” particle energy arises which determines the frequency of spin precession in three-dimensional space. Thus the problem of the maximum spin coherence time is reduced to the minimization of the particles’ orbit length dispersion. This is achieved by introducing sextupoles with fields minimizing the chromaticity of spin precession frequency. We have experimentally found that minimizing the chromaticity of betatron motion leads to the minimization of the chromaticity of cyclotron frequency.

1. **Three components of deviation from the axis determining orbit lengthening**

Let us write the motion equation in the horizontal and vertical planes  with the longitudinal coordinate :

(1)

where is the quadrupole strength with gradient, is the orbit curvature, the momentum spread, and are external nonlinear forces of an arbitrary order. In particular, for the sextupole field: and , where with the gradient . Here we introduce the signifier “kick” since further on we will consider the right hand side of equation (1) as a weak perturbation of a free oscillation of particle concentrated on a short period with fixed deviation from axis.

The total displacement of a non-referent particle relative of the referent one is defined through three components:

,

. (2)

1. ***The “-oscillating” term***

We assume that the dispersion in the vertical plane is absent. Now let us define each term individually. We start with the first-oscillating term . It can be determined by solving the homogeneous Hill’s equation describing free oscillation in a zero-dispersion optics:

, (2a)

and using Floquet theorem [1]:

, (3)

where is the beta function, is Floque phase, is an action integral. Instead of an action integral an emittance is often used: . For the vertical plane it will be similar

. (3a)

1. ***The “dispersion” term***

Let us now define the term arising due to the dispersion in the horizontal plane, where the periodical solution of dispersion is determined from the equation (without ):

(4)

The periodic solution of (4) is sought using the method of variation of constants:

. (4a)

Multiplying (4a) by the momentum spread we obtain the sought-after second term of (2) :

(5)

1. ***The “external kick” term***

Here we assume that at an arbitrary point on the orbit we have a kick-value . Let us again use the method of variation of constants. By integrating along the element length in a constant kick value approximation, and summing over all elements, we have the expression for :

(6)

1. **Orbit lengthening**

We have all the necessary expressions to determine the orbit lengthening. Commonly it is defined as

, (7)

where are the total deviations in both coordinates.

1. ***The non-linear contribution***

Let us start with the non-linear contribution entering into the second integral of (7).

Using (3) and the ratio for the Twiss parameters and we have for the homogeneous equation (2a)

(8)

Since the average value over whole ring , we have and and hence (8) is:

(9)

Accordingly for the vertical plane:

. (10)

Substituting (9,10) in (7) we have:

(11)

1. ***The linear contribution***

The contribution of the -oscillating term in the first part of (7) will be zero; therefore, the first part of (7) is defined only by

, (12)

and

. (13)

Since the integral in (13) is the definition of the momentum compaction factor:

(14)

Obviously, a significant contribution to the orbit-lengthening in (13) is due to the linear term  of the momentum compaction (14). To compensate for the effect of linear dependence of the orbit lengthening on the momentum spread RF field is used which mixes by energy particles with different deviations from the equilibrium momentum thus obtaining . However it does not allow getting ; hence, in the case of a significant contribution from this term, it will be necessary to adjust the optics using sextupoles up to .

1. ***External kick***

Using expression (6) and substituting it into we have the orbit lengthening due to a non-linear external kick:

. (15)

Thus the orbit lengthening due to an external kick is equal to the sum over optic elements of the product of the normalized kick and the dispersion function in the point-of-kick. In the next section this formula will be applied to the non-linear sextupole kick.

1. **Sextupole correction**

Now let us consider the sextupole correction. In that case the equation (1) takes the form:

, (16)

Many authors simplify the solution of this equation by substituting the nonlinear term by a constant kick along *i-th* sextupole with a force

(17)

proportional to the square of the particle’s deviation from the axis; that is by replacing the nonlinear term by a constant one. Substituting (3,3a) in (17) we have:

. (18)

Following (15) we can write the expression for the orbit lengthening due to sextupoles:

(19)

Or, passing to integration in (19), we have the total orbit lengthening together with (11) and (14):

. (20)

1. **Chromaticity and orbit lengthening**

The chromaticity in both planes is defined by a well-known formula [2]:

. (21)

Simultaneously, from the Twiss relations obtains

, (22)

since . Finally, we have the expression for chromaticity:

. (23)

Comparing (20) with (23) we can rewrite the orbit lengthening through the chromaticity:

. (24)

1. **Conclusion**

Thus we can say that for a relatively short sextupole length betatron oscillations make a zero contribution to the spin chromaticity if the betatron chromaticity is zero. However, the longitudinal motion contributes to the spin chromaticity independently of the betatron chromaticity, and its contribution can be reduced to zero at presence of an RF field and a zero second-order momentum of the compaction factor .

1. **References**
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